HDP-003-0011005 Seat No. _____

B. Sc. (Sem. I) (CBCS) Examination

November/December – 2017

MATH.-01(A) : Calculus

Faculty Code : 003
Subject Code : 0011005

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Figures to the right indicate full marks of the question.

1 (a) Answer the following : **4**

(1) State the Rolle's Theorem.

(2) What is the geometric interpretation of Rolle's Theorem ?

(3) Why Lagrange's Theorem is not applicable to the function $f(x) = x^{\frac{2}{3}}$ $x \in [-2, 2]$?

(4) Write the Maclaurian's series expansion of sin x up to 5 terms.

(b) Attempt any one : **2**

(1) Obtain the Maclaurian's series expansion of 5^x up to first three non-zero terms.

(2) Expand $\log(1+x)$ in powers of x up to first four non-zero terms.

(c) Attempt any one : **3**

(1) If $f(x) = \sqrt{x}$ and $g(x) = \frac{1}{\sqrt{x}}$ for $x \in [a, b] (a > 0)$, then prove that C of Cauchy's Mean Value Theorem is geometric mean between a and b.

(2) Verify Rolle's Theorem for $f(x) = e^x(\sin x - \cos x)$ in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.

(d) Attempt any one : **5**

(1) Prove that $2x^3 - 3x^2 - x + 1 = 0$ has at least one real root between 1 and 2.

(2) State and prove Lagrange's Mean Value Theorem.

2 (a) Answer the following : **4**

(1) Write seven indeterminant forms (e.g. $\frac{0}{0}$ is one of the seven indeterminant forms)

(2) Define Homogeneous function.

(3) Write the general form of first order and first degree linear differential equation.

(4) Cheek whether $y' = \frac{x^2 - y^2}{2x}$ is homogeneous differential equation or not.

(b) Attempt any one : **2**

(1) Evaluate : $\lim_{x \to 0} \frac{xe^x - \log(1+x)}{x^2}$.

(2) Solve : $\frac{dy}{dx} = e^{x-y} + x^2 e^{-y}$.

(c) Attempt any one : **3**

(1) Prove that $\lim_{x \to 1} (1 - x^2) \frac{1}{\log(1-x)} = e$.

(2) Solve : $(x \log x) y' + y = 2 \log x$.

(d) Attempt any one : **5**

(1) If $\lim_{x \to 0} \frac{\sin 2x + p \sin x}{x^3}$ is finite, then find the value of p and hence evaluate the limit.

(2) Solve : $2ye^{\frac{x}{y}}dx + \left(y - 2xe^{\frac{x}{y}} \right) dy = 0$.

3 (a) Answer the following : **4**

(1) Define Bernoulli's differential equation.

(2) Cheek whether the differential equation $xdy + ydx + 2x^3dx = 0$ is exact or not.

(3) Give an example of a Clairaut's differential equation.

(4) Find integrating factor of the differential equation $x\frac{dy}{dx} - y = 2x^2$.

(b) Attempt any one : **2**

(1) Solve : $y = xp - p^2 + \log p$, Where $p = \frac{dy}{dx}$.

(2) Solve : $\left(y^2 - x^2 \right) dx + 2xy\, dy = 0$.

(c) Attempt any one : **3**

(1) Solve : $\left(y^4 + 2y \right) dx + \left(xy^3 + 2y^4 - 4x \right) dy = 0$.

(2) Solve : $\left(2xy \cos x^2 - 2xy + 1 \right) dx + \left(\sin x^2 - x^2 \right) dy = 0$

(d) Attempt any one : **5**

(1) Solve : $\left(2xy^4 e^y + 2xy^3 + y \right) dx + \left(x^2 y^4 e^y - x^2 y^2 - 3x \right) dy = 0$

(2) Solve : $p^2 - 6px + 3y = 0$, where $p = \frac{dy}{dx}$.

4 (a) Answer the following : **4**

(1) Give an example of an homogeneous higher order linear differential equation.

(2) Solve : $D^2y + 6Dy + 9y = 0$, where $D = \frac{d}{dx}$.

(3) Solve : $\left(D^4 + 4D^2\right)y = 0$, where $D = \frac{d}{dx}$.

(4) If the roots of the auxiliary equation are 1, 1, 2, 2, then what is the C.F. ?

(b) Attempt any one : **2**

(1) Solve : $\left(D^4 + 4\right)y = 0$, where $D = \frac{d}{dx}$.

(2) Solve : $\left(D^3 - D^2 + 4D - 4\right)y = e^x$, where $D = \frac{d}{dx}$.

(c) Attempt any one : **3**

(1) Solve : $\left(D^2 + 6D + 9\right)y = 5^x - \log 2$, where $D = \frac{d}{dx}$.

(2) In usual notation prove that :

$$\frac{1}{f(D)}e^{ax}V(x) = e^{ax}\frac{1}{f(D+a)}V(x).$$

(d) Attempt any one : **5**

(1) Solve : $\left(D^3 + 3D^2 - 4D - 12\right)y = 12xe^{-2x}$, where

$D = \frac{d}{dx}$.

(2) Solve : $\left(D^3 - 3D^2 + 4D - 2\right)y = e^x + \cos x$ where

$D = \frac{d}{dx}$.

5 (a) Answer the following : **4**

(1) Evaluate : $\frac{1}{(D-1)^2}e^x$.

(2) Evaluate : $\frac{1}{D+1}2x$.

(3) Find P.I. of $\left(D^2-7D+6\right)y=e^{2x}$.

(4) Solve : $\left(4x^2D^2+1\right)y=0$, where $D=\frac{d}{dx}$.

(b) Attempt any one : **2**

(1) Solve : $\left(4x^2D^2+16xD+9\right)y=0$, where $D=\frac{d}{dx}$.

(2) Solve : $\left(x^2D^2+xD-1\right)y=0$, where $D=\frac{d}{dx}$.

(c) Attempt any one : **3**

(1) Solve : $\left(x^3D^2+x^2D\right)y=x^6+x^4+2x$, where $D=\frac{d}{dx}$.

(2) Solve : $\left((3x+2)^2D^2+3(3x+2)D-36\right)y=3x^2+4x+1$,

where $D=\frac{d}{dx}$.

(d) Attempt any one : **5**

(1) Solve : $\left(x^2D^2 - xD + 2\right)y = 6,\ y(1) = 1,\ y'(1) = 2$,

where $D = \frac{d}{dx}$.

(2) Solve : $\left(4x^2D^2 + 1\right)y = \log x,\ x > 0,\ y(1) = 0,\ y(e) = 5$,

where $D = \frac{d}{dx}$.